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Rational Possibility of Generating Power Laws in the Synthesis of Cam Mechanisms

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Introduction.The generation of polynomial power laws of motion for the synthesis of cam mechanisms is complicated by the need to determine the coefficients of power polynomials. The study objective is to discover a rational capability of generatingpowerlawswitharbitrarytermsnumberunderthesynthesisofcammechanisms.
Materials and Methods.A unified formula for determining the values of coefficients of power polynomials with any number of integers and/or non-integer exponents is derived through the so-called transfinite mathematical induction.
Results.A unified formula for determining the values of coefficients, which gives correct results for any number of even and/or odd exponents, is presented. The correctness of the derived formula is validated by the results on the multiple checks for different numbers, even and odd values of the exponents of quinquinomial and hexanomial power functions.
Discussion and Conclusions. A unified formula for determining the values of coefficients of power polynomials makes it possible to rationally define the laws of motion without finite and infinite spikes in the synthesis of elastic cam-lever systems. This provides a rational determination of the laws of motion without finite and infinite spikes in the synthesis of elastic cam-lever systems, and simple verification of the accuracy of the results obtained. The functions are particularly suitable for the synthesis of polydyne cams, as well as cams, since one polynomial can be used throughout the entire geometric mechanism cycle.

Keywords: cam mechanisms, laws of motion, power functions.

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1. Introduction

One of the most important tasks in the design of cam mechanisms [1−4] and in theplanning of industrial robots movements [5−7], is undoubtedly the selection of the law of motion, as the law affects the basic kinematic, force and dynamic characteristics of the generated movements [8−11].

It is generally assumed that the units are rigid bodies connected without a gap clearance, whereby the mechanism generates the desired basic law of motion. In fact, real laws of motion of the mechanisms differ significantly from the basalones as the speed of the cam, the load, the deformations, and the clearances of the cam-lever systems are greater.

The cams, synthesized according to polynomial laws of motion taking into account the dynamics and deformations of the mechanical system driven by the cam, are called polydyne cams.The design of such cams is required for the construction of high-speed and insufficiently rigid mechanical systems.

The development of methods for the synthesis of polydyne cams was started in 1948 by Dudley [12], supplemented and developed by many other authors mainly in connection with dynamic studies of cam-lever systems [13−18]. The main purpose of the methods is to exclude the acceleration breaks (jerks), resp. of the inertial load of resiliently susceptible mechanical systems to achieve more precise target movements with minimum oscillations.

The design of polydyne cams is required not only for cam-lever valves of automobile engines [17−19], but also for many other high-speed and insufficiently rigid mechanical systems of various technological machines [15], [19−22].

Power-polynomial laws of motion with four or more terms have great advantages in achieving the desired boundary conditions at the beginning and at the end of the phases of movement of the output at the cam mechanisms [15], [20, 21]. Such motion laws are suitable for the synthesis of mechanisms with polydyne cams [1−3], [5]. These laws make providemodelingthe laws of motion without finite and infinite spikes with better dynamic characteristics of high-speed, elastic cam-lever systems than the power trinomial and quadrinomial laws of motion. However, the derivation of power laws of motion with four or more terms is difficult due to the need to solve systems with four and more equations, respectively.

The aim of the study is to explorea rational possibility for generating basic power laws with arbitrary number of terms when formulating design laws of motion for the synthesis of cam mechanisms.

2. Materials and Methods

Thebasal law of motion of polydynamic cam mechanisms is most significantly affected by the basal second transfer function and its derivatives. This function, multiplied by the dynamic constant of the cam-driven mechanical system, changes the output displacement, as the inertial load generated by the acceleration deforms the system components elastically.In other words, the second derivative (the basal second transfer function) also participates in the real displacement function.

Therefore, in order to avoid spikes in the first two real transfer functions, it is required to avoid spikesin the next two basal transfer functions — the third and the fourth.This cannot be achieved for the limits of the phases of movement of the output unit if a power trinomial or quadrinomial displacement function is selected. These spikes will be avoided if the displacement function and its first four derivatives are continuous functions.

The displacement function of the output link of the cam mechanism may, in any law of motion, be written in summary form $B = B_0 + \Delta B(\varphi) = B_0 + H.u(\xi)$, where B is the output coordinate formed by its initial value B_0, which determines the initial position of the output link, to which the *displacement function* of the output link is added — a product of the follower motion $H \equiv \Delta B_{\max}$ and the normalized function $u(\xi)$. The velocity, acceleration and the subsequent derivative (jerk) of the follower's motion correspond to the transfer functions $B'(\varphi)$, $B''(\varphi)$, $B'''(\varphi)$,which differ by only one factor H/Φ_1, H/Φ_1^2 and H/Φ_1^3 (Φ_1 is the cam angle of the follower rise) respectively from the derivatives u', u'' and u''' of the normalized function $u(\xi) \in [0;\ 1]$ in the argument $\xi = \varphi / \Phi_1 \in [0;\ 1]$:

$$\begin{cases} \Delta B = H\,.\,u(\xi); \\ B' = \dfrac{H}{\Phi_1} u'(\xi); \\ B'' = \dfrac{H}{\Phi_1^{\,2}} u''(\xi); \\ B''' = \dfrac{H}{\Phi_1^{\,3}} u'''(\xi)... \end{cases} \tag{1}$$

For a ***binomial power function*** with the exponents k and m, the coefficients a_k and a_m are determined by the relations:

$$a_k = \frac{m}{(m-k)}; \quad a_m = \frac{k}{(k-m)}.$$

For a ***trinomial power function*** with the exponents k, m and p, the coefficients a_k, a_m and a_p are determined by the relations:

$$a_k = \frac{m\ p}{(m-k)(p-k)}; \quad a_m = \frac{k\ p}{(k-m)(p-m)}; \quad a_p = \frac{k\ m}{(k-p)(m-p)}$$

There are known formulas for determining the coefficients of normalized power functions up to four integers and/or non-integer power exponents.

For a ***quadrinomial power function*** with the exponents k, m, p and q, the coefficients a_k, a_m, $a_p$and a_q are determined by the relations:

$$a_k = \frac{m\ p\ q}{(m-k)(p-k)(q-k)};\ a_m = \frac{k\ p\ q}{(k-m)(p-m)(q-m)};$$

$$a_p = \frac{k\ m\ q}{(k-p)(m-p)(q-p)};\ a_q = \frac{k\ m\ p}{(k-q)(m-q)(p-q)}.$$

Aformula for determining the coefficients of normalized power functions with an arbitrary number of integer and non-integer exponents is derived.

According to the method of the so-called *transfinite mathematical induction*, it can be assumed that the formulas for determining the values of the coefficients of the input normalized power functions are valid for any plurality of integer and non-integer exponents. The known formulas for determining the values of the coefficients are true for two, three and four even and odd exponents, from which the ***inductive assumption***follows that for any number of even and/or odd exponents, a formula for the values of the coefficients is inductively obtained

$$a_j = \frac{k\ .\ m\ .\ p\ ...\ v}{(k-j)(m-j)(p-j)...(v-j)}\ , \qquad (2)$$

in which j consistently takes n in the number of values $k, m, p, ..., v$. The numerator of (2) excludes the exponent j (it isassumed that j = 1), and in the denominators of any value of exponents (except j), the value of j is subtracted. In other words, the value of each unknown coefficient a_j of the normalized power function is determined by the relation (2) with the numerators, which isthe product of the exponents, excluding j,and the denominator, which is the product of the difference between the exponents (except j) and the exponent j.

3. Results

To verify the results obtained, the sum of the values of the calculated coefficients must be equal to one:

$$k + m + p + ... + v = 1.$$

An ***inductive inference*** for (2) is reached if it is also proved that an arbitrary number $n$is odd and/or odd values of exponents. The correctness of formula (2) is validated by the results on the multiple checks for different numbers of n, even and odd values of the exponents of quinquinomial and hexanomial power functions. Two functions have been selected from them.

Example 1. Let the power function be quinquinomial with integers and non-integers exponents. For example, at k = 5; m = 5.5; p = 6;q = 6.5; s = 7 from formula (2), it is obtained:

a_k = 1001; a_m = -3640; a_p = 5005; a_q = -3080; a_s = 715.

The results are true since$a_k + a_m + a_p + a_q + a_s = 1$.

Thus, for the normalized power function and its derivatives, we obtain:

$$\begin{cases} u = 1001\xi^5 - 3640\xi^{5.5} + 5005\xi^6 - 3080\xi^{6.5} + 715\xi^7, \\ u' = 5005(\xi^4 - 4\xi^{4.5} + 6\xi^5 - 4\xi^{5.5} + \xi^6), \\ u'' = 10010(2\xi^3 - 9\xi^{3.5} + 15\xi^4 - 11\xi^{4.5} + 3\xi^5), \\ u''' = 15015(4\xi^2 - 21\xi^{2.5} + 40\xi^3 - 33\xi^{3.5} + 10\xi^4), \\ u'''' = 15015(8\xi - 52.5\xi^{1.5} + 120\xi^2 - 115.5\xi^{2.5} + 40\xi^3) \end{cases} \qquad (3)$$

Indeed, for the interval boundaries $\xi \in [0, 1]$, the function $u(\xi)$ has values of 0 and 1, respectively, and all derivatives functions of $u(\xi)$ are zeroing.

Figure1presents the power polynomial $u(\xi)$ with the first three derivatives.

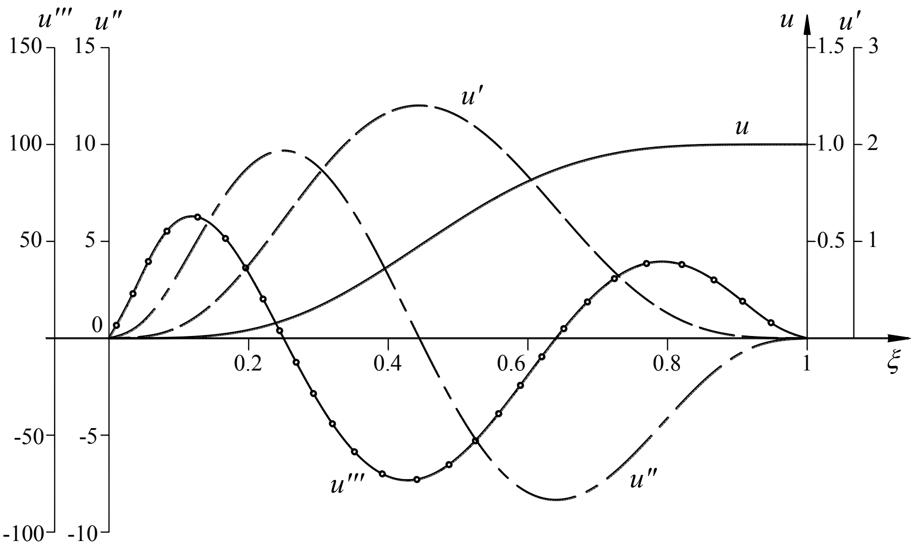

Fig. 1. Graphs of the quinquinomial u(ξ) and the first three derivatives by equation (3)

Example 2. Let the power function be hexanomial with integer power, for example:

k = 6; m = 7; p = 8; q = 9; s = 10; v = 11. From formula (2),we obtaina_k = 462; a_m = -1980; a_p = 3465; a_q = -3080; a_s = 1386; a_v = -252,and therefore:

$$\begin{cases} u = 462\xi^6 - 1980\xi^7 + 3465\xi^8 - 3080\xi^9 + 1386\xi^{10} - 252\xi^{11}, \\ u' = 2772(\xi^5 - 5\xi^6 + 10\xi^7 - 10\xi^8 + 5\xi^9 - \xi^{10}), \\ u'' = 13860(\xi^4 - 6\xi^5 + 14\xi^6 - 16\xi^7 + 9\xi^8 - 2\xi^9), \\ u''' = 27720(2\xi^3 - 15\xi^4 + 42\xi^5 - 56\xi^6 + 36\xi^7 - 9\xi^8), \\ u''''= 166320(\xi^2 - 10\xi^3 + 35\xi^4 - 56\xi^5 + 42\xi^6 - 12\xi^7), \\ u''''' = 332640(\xi - 15\xi^2 + 70\xi^3 - 140\xi^4 + 126\xi^5 - 42\xi^6). \end{cases} \tag{4}$$

Expectedly, for the boundaries of the interval $\xi \in [0, 1]$, the function $u(\xi)$ has values of 0 and 1, respectively, and all derivatives functions of $u(\xi)$ by the fifth line are zeroing. This means that the polynomial has one common point and 5 infinitely close common points with the axis ξ at $\xi = 0$ and $\xi = 1$ in the positive direction to the axis ξ and another 5 infinitely close common points with the axis ξ at $\xi = 0$ and $\xi = 1$ in the opposite direction to the axis ξ. In practice, this means 11 infinitely close common points of the polynomial with the ξ axis or an oscillation (tangent) of 10 lines of the polynomial with the ξ axis. Although infinitely close, the common points generally lead to an approximate, but sufficiently accurate, in some cases dwell of the output link. Figure 2 presents the power polynomial $u(\xi)$ with the first three derivatives.

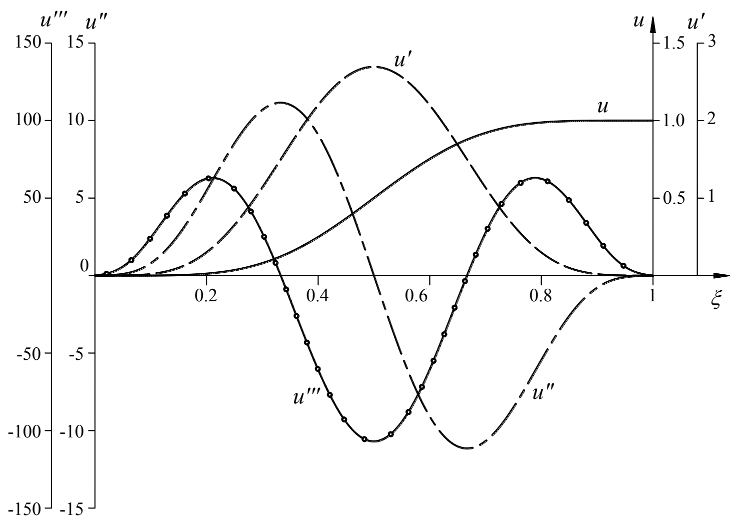

Fig. 2. Graphs of the hexanomial polynomial u(ξ) and the first three derivatives by equation (4)

For values of the exponents k = 7; m = 8; p = 9; q = 10; s = 11; v = 12 from formulas (2), it is obtained: a_k = 792; a_m = -3465; a_p = 6160; a_q = -5544; a_s = 2520; a_v = -462.

Then the normalized function and its derivatives are specified in the form:

$$\begin{cases} u = 792\xi^7 - 3465\xi^8 + 6160\xi^9 - 5544\xi^{10} + 2520\xi^{11} - 462\xi^{12}, \\ u' = 5544(\xi^6 - 5\xi^7 + 10\xi^8 - 10\xi^9 + 5\xi^{10} - \xi^{11}), \\ u'' = 5544(6\xi^5 - 35\xi^6 + 80\xi^7 - 90\xi^8 + 50\xi^9 - 11\xi^{10}), \\ u''' = 55440(3\xi^4 - 21\xi^5 + 56\xi^6 - 72\xi^7 + 45\xi^8 - 11\xi^9), \\ u''''= 166320(4\xi^3 - 35\xi^4 + 112\xi^5 - 168\xi^6 + 120\xi^7 - 33\xi^8), \\ u''''' = 665280(3\xi^2 - 35\xi^3 + 140\xi^4 - 252\xi^5 + 210\xi^6 - 66\xi^7), \\ u'''''' = 665280(6\xi - 105\xi^2 + 560\xi^3 - 1260\xi^4 + 1260\xi^5 - 462\xi^6). \end{cases} \tag{5}$$

The check $a_k + a_m + a_p + a_q + a_s + a_v = 1$, the normalized function $u(\xi)$, and its derivatives show the accuracy of the results obtained, because of the interval's boundaries $\xi \in$ [0; 1], the function $u(\xi)$ has values of 0 and 1, respectively, and all derivative functions are zeroing.

The graphs of $u(\xi)$, $u'(\xi)$, $u''(\xi)$, and $u'''(\xi)$ are presented in Figure 3, which shows that at the beginning and at the end of the cam angle Φ_1 in the rise phase distance phase (rise phase, outstroke phase), the follower remains practically stationary — an approximate dwell of the follower is realized. In the cam angle Φ_3 in the return phase (reverse move, return stroke) of the follower, the normalized power functions $u(\underline{\xi})$, $u'(\underline{\xi})$, $u''(\underline{\xi})$ and $u'''(\underline{\xi})$ are retained by type with a new argument $\underline{\xi} = 1 - \xi$, where $\xi \in$ [0, 1].

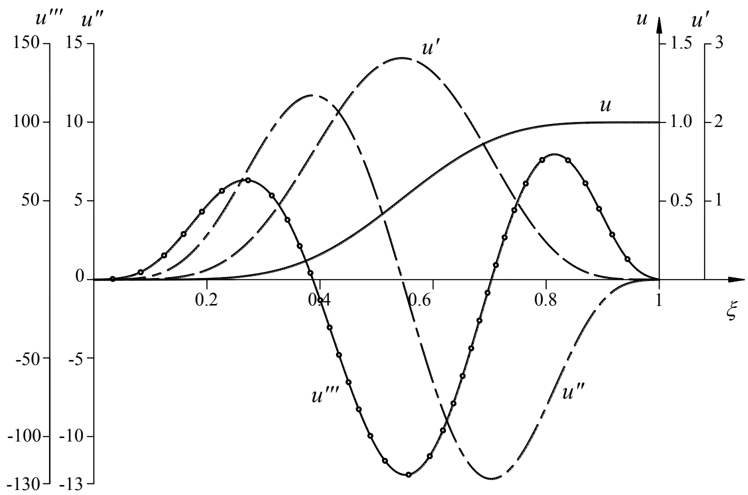

Fig. 3. Graphs of the hexanomial u(ξ) and the first three derivatives by equation (5)

A detailed solution to the question of the laws of motion and synthesis of cam mechanisms was made by Galabov, Roussev, and Paleva-Kadiyska in [4].

4. Discussion and Conclusions

The spikesin the first two transfer functions in high-speed, resilient (elastic) cam-lever systems are avoided if the displacement function and its first four derivatives are continuous functions.This cannot be achieved for the limits of the phases of movement of the output unit if a power trinomial or quadrinomial displacement function is selected. However, the derivation of power laws of motion with four or more terms is complicated by the need to solve systems with four or more equations, respectively. Therefore, by the method of the so-called transfinite mathematical induction, a unified formula for determining the values of coefficients of power polynomials with any number of integers and/or non-integer exponents is derived. It gives a rational possibility for defining the laws of motion without finite and infinite spikes in the synthesis of elastic cam-lever systems and easy verification of the accuracy of the results obtained.

The functions (3), (4), (5) and other polynomial power polynomials are especially suitable for the synthesis of polydyne cams, as well as cams, since one polynomial can be used throughout the geometric mechanism cycle.

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Claimed contributorship:

B. I. Paleva-Kadiyska: preparation of the text; preparation of theresults and graphs;formulation of conclusions; translation of the paper into English; translation of the abstract and keywords into Russian.R. A. Roussev:review of literature sources;calculations; computational analysis; analysis of the research results.V. B. Galabov: basic concept formulation; formulation of the research purpose and tasks;academic advising; text processing; correction of the conclusions;approval of the final version of the paper before submitting it for publication.

All authors have read and approved the final manuscript.